Mapped Mesh Generator
Function　Manual

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1. Curved surface interpolation

The interpolation curved surface is given by the following formula by the use of the boundary point. In the finite element method, it is used as a shape function.

$$S(u,v) = N_0 p_0 + N_1 p_1 + N_2 p_2 .....N_n p_n$$

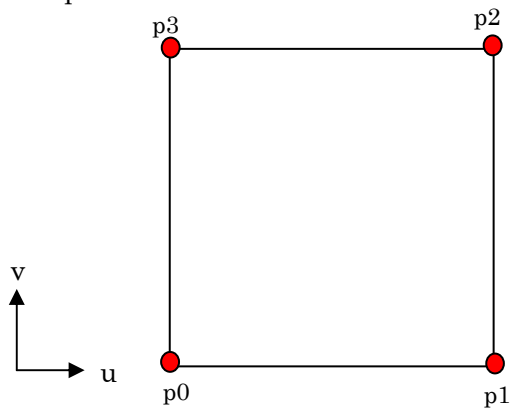
- N_n is an interpolation coefficient.
- p_n is control point.
- The range of parameter (u and v)is $0 \le u \le 1$, $0 \le v \le 1$.

1.1 Bilinear surface

1.1.1 Formula

The bilinear surface is representation using four points. In the finite element method, it is used as a shape function.

$$N0 = \frac{1}{4}(1-u)(1-v)$$
$$N0 = \frac{1}{4}(1+u)(1-v)$$
$$N1 = \frac{1}{4}(1+u)(1+v)$$
$$N2 = \frac{1}{4}(1-u)(1+v)$$



Bilinear surface (4 points)

The range of parameter of shape function for finite element method is representation as $-1 \le u \le 1$, $-1 \le v \le 1$.

The range of parameter (u and v)can be changed to $0 \le u \le 1$, $0 \le v \le 1$ by the following formula.

$$u = (u' \times 2.0) - 1.0$$
$$v = (v' \times 2.0) - 1.0$$

1.1.2 Alignment of edge

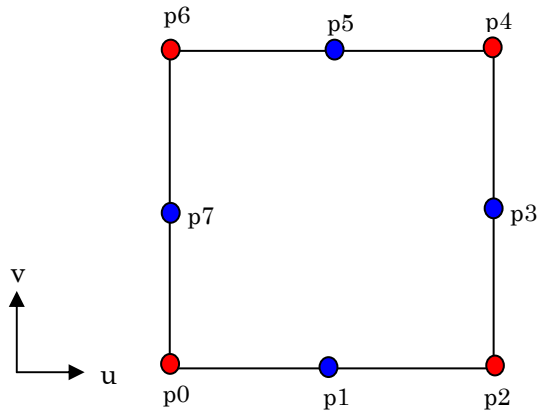
No	Number of edge	Number of point that composes the edge
1	e0	p0,p1
2	e1	p1,p2
3	e2	p2,p3
4	e3	p3,p0

1.2 Quadric surface

1.2.1 Formula

The quadric surface is representation using 8 points.

$$N0 = (1-u)(1-v)(1-2u-2v)$$
$$N1 = 4u(1-u)(1-v)$$
$$N2 = u(1-v)(2u-2v-1)$$
$$N3 = 4uv(1-v)$$
$$N4 = uv(2u+2v-3)$$
$$N5 = 4u(1-u)v$$
$$N6 = (1-u)v(2v-2u-1)$$
$$N7 = 4(1-u)v(1-v)$$



quadric surface (eight points)

1.2.2 Alignment of edge

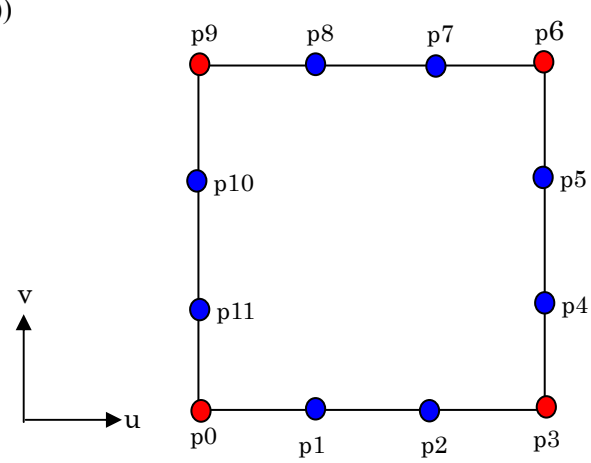
No	Id of edge	Id of point that composes the edge
1	e0	p0,p1,p2
2	e1	p2,p3,p4
3	e2	p4,p5,p6
4	e3	p6,p7,p0

1.3 Cubic Surface

1.3.1 Formula

The cubic surface is representation using 12 points.

$$N0 = 0.5(1-u)(1-v)(2-9u(1-u)-9v(1-v))$$
$$N1 = 4.5u(1-u)(2-3u)(1-v)$$
$$N2 = -4.5u(1-u)(1-3u)(1-v)$$
$$N3 = 0.5u(1-v)(2-9u(1-u)-9v(1-v))$$
$$N4 = 4.5uv(1-v)(2-3v)$$
$$N5 = -4.5uv(1-v)(1-3v)$$
$$N6 = 0.5uv(2-9u(1-u)-9v(1-v))$$
$$N7 = -4.5u(1-u)(1-3u)v$$
$$N8 = 4.5u(1-u)(2-3u)v$$
$$N9 = 0.5(1-u)v(2-9u(1-u)-9v(1-v))$$
$$N10 = -4.5(1-u)v(1-v)(1-3v)$$
$$N11 = 4.5(1-u)v(1-v)(2-3v)$$



Cubic surface(twelve points)

1.3.2 Alignment of edge

No	Id of edge	Id of point that composes the edge
1	e0	p0,p1,p2,p3
2	e1	p3,p4,p5,p6
3	e2	p6,p7,p8,p9
4	e3	p9,p10,p11,p0

2. volume interpolation

The volume interpolation is given by the following formula by the use of the boundary point. In the finite element method, it is used as a shape function.

$$V(u,v,t) = N_0 p_0 + N_1 p_1 + N_2 p_2 .....N_n p_n$$

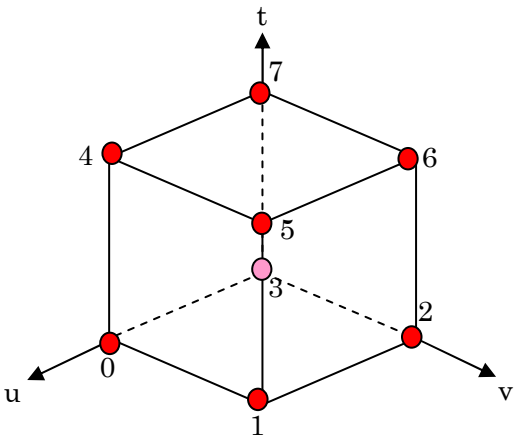
- N_n is an interpolation coefficient.
- p_n is control point.
- The range of parameter (u and v and t)is $0 \le u \le 1$, $0 \le v \le 1$, $0 \le t \le 1$.

2.1 Bilinear hexahedron

2.1.1 Formula

The bilinear hexahedron is representation using eight points. In the finite element method, it is used as a shape function.

$$N0 = (1+u)(1-v)(1-t)$$
$$N1 = (1+u)(1+v)(1-t)$$
$$N2 = (1-u)(1+v)(1-t)$$
$$N3 = (1-u)(1-v)(1-t)$$
$$N4 = (1+u)(1-v)(1+t)$$
$$N5 = (1+u)(1+v)(1+t)$$
$$N6 = (1-u)(1+v)(1+t)$$
$$N7 = (1-u)(1-v)(1+t)$$



Bilinear hexahedron(eight points)

The range of parameter of shape function for finite element method is representation as $-1 \le u \le 1$, $-1 \le v \le 1$, $-1 \le t \le 1$.

The range of parameter (u and v and t)can be changed to $0 \le u \le 1$, $0 \le v \le 1$, $0 \le t \le 1$ by the following formula.

$$u = (u' \times 2.0) - 1.0$$
$$v = (v' \times 2.0) - 1.0$$
$$t = (t' \times 2.0) - 1.0$$

2.1.2 Alignment edge and face

(1)Edge

No	Id of edge	Id of point that composes the edge
1	e0	p0,p1
2	e1	p1,p2
3	e2	p2,p3
4	e3	p3,p0
5	e4	p0,p4
6	e5	p1,p5
7	e6	p2,p6
8	e7	p3,p7
9	e8	p4,p5
10	e9	p5,p6
11	e10	p6,p7
12	e11	p7,p4

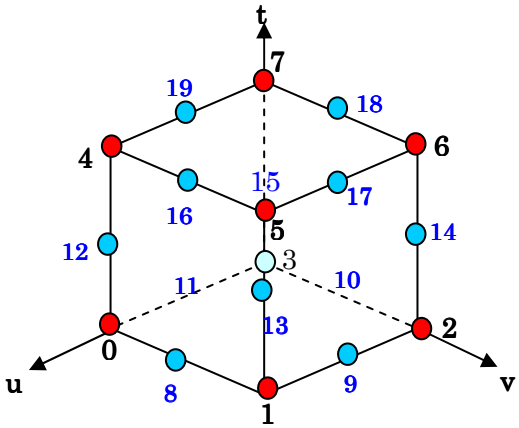
(2)Face

No	Id of edge	Id of point that composes the face
1	f0	p0,p3,p2,p1
2	f1	p4,p5,p6,p7
3	f2	p0,p1,p5,p4
4	f3	p1,p2,p6,p5
5	f4	p2,p3,p7,p6
6	f5	p0,p4,p7,p3

2.2 Quadric hexahedron

2.2.1 Formula

The quadric hexahedron is representation using 20 points. In the finite element method, it is used as a shape function.



$$N0 = \frac{1}{8}(1-u)(1-v)(1-t)(-2-u-v-t) \qquad N10 = \frac{1}{4}(1-u^2)(1+v)(1-t)$$

$$N1 = \frac{1}{8}(1+u)(1-v)(1-t)(-2+u-v-t) \qquad N11 = \frac{1}{4}(1-u)(1-v^2)(1-t)$$

$$N2 = \frac{1}{8}(1+u)(1+v)(1-t)(-2+u+v-t) \qquad N12 = \frac{1}{4}(1-u)(1-v)(1-t^2)$$

$$N3 = \frac{1}{8}(1-u)(1+v)(1-t)(-2-u+v-t) \qquad N13 = \frac{1}{4}(1+u)(1-v)(1-t^2)$$

$$N4 = \frac{1}{8}(1-u)(1-v)(1+t)(-2-u-v+t) \qquad N14 = \frac{1}{4}(1+u)(1+v)(1-t^2)$$

$$N5 = \frac{1}{8}(1+u)(1-v)(1+t)(-2+u-v+t) \qquad N15 = \frac{1}{4}(1-u)(1+v)(1-t^2)$$

$$N6 = \frac{1}{8}(1+u)(1+v)(1+t)(-2+u+v+t) \qquad N16 = \frac{1}{4}(1-u^2)(1-v)(1+t)$$

$$N7 = \frac{1}{8}(1-u)(1+v)(1+t)(-2-u+v+t) \qquad N17 = \frac{1}{4}(1+u)(1-v^2)(1+t)$$

$$N8 = \frac{1}{4}(1-u^2)(1-v)(1-t) \qquad N18 = \frac{1}{4}(1-u^2)(1+v)(1+t)$$

$$N9 = \frac{1}{4}(1+u)(1-v^2)(1-t) \qquad N19 = \frac{1}{4}(1-u)(1-v^2)(1+t)$$

The range of parameter of shape function for finite element method is representation as $-1 \le u \le 1$, $-1 \le v \le 1$, $-1 \le t \le 1$.

The range of parameter (u and v and t)can be changed to $0 \le u \le 1$, $0 \le v \le 1$, $0 \le t \le 1$ by the following formula.

$$u = (u' \times 2.0) - 1.0$$

$$v = (v' \times 2.0) - 1.0$$

$$t = (t' \times 2.0) - 1.0$$

2.2.2 Alignment edge and face

(1)Edge

No	Id of edge	Id of point that composes the edge
1	e0	p0,p8,p1
2	e1	p1,p9,p2
3	e2	p2,p10,p3
4	e3	p3,p11,p0
5	e4	p0,p12,p4
6	e5	p1,p13,p5
7	e6	p2,p14,p6
8	e7	p3,p15,p7
9	e8	p4,p16,p5
10	e9	p5,p17,p6
11	e10	p6,p18,p7
12	e11	p7,p19,p4

(2)Face

No	Id of edge	Id of point that composes the face
1	f0	p0,p11,p3,p10,p2,p9,p1,p8
2	f1	p4,p16,p5,p17,p6,p18,p7,p19
3	f2	p0,p8,p1,p13,p5,p16,p4,p12
4	f3	p1,p9,p2,p14,p6,p17,p5,p13
5	f4	p2,p10,p3,p15,p7,p18,p6,p14
6	f5	p0,p12,p4,p19,p7,p15,p3,p11